

NONEQUILIBRIUM KINETIC TRANSITIONS IN SOLIDS AS MECHANISMS OF LOCAL
PLASTIC STRAIN LOCALIZATION

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Plastic strain localization has been detected for a long time. However, it is assumed that this phenomenon is only typical for high levels of shape change for a solid. Recently it has been established that any inelastic strain does not occur uniformly and different regions of deformed material are involved in turn in the process, whereas in other parts plastic flow is weakly expressed [1-6]. The effects of plastic instability occur both with simple (tension [7], compression [8], torsion [9]), and with complex (forging, stamping, rolling, etc. [10]) loading regimes. Analysis of experimental results [11, 12] leads to the conclusion that the capacity of polycrystalline materials for plastic flow is a capacity for strain localization. Thus, an important problem is creating physical substantiation for describing the deformation of solids taking account of plastic strain localization.

In the present work the behavior of specimens of ductile polycrystalline materials under uniaxial tensile conditions is considered. According to data in [13] strain localization in this case develops in two stages. First in the form of a 'running' neck, i.e., simultaneous occurrence of stabilization of local thinning in a specimen. Subsequently, with achievement of strain for reduction of the order of 0.15-0.25 a stable neck develops continuously reducing its deforming volume to failure.

Numerous studies of plastic strain localization indicate that this may be accomplished by a marked thermal effect and development of structural and phase transformations governing a set of physicomechanical material properties [14, 15]. In experiments this develops as a reduction in deformation resistance, changes in strengthening coefficients, and an anomalous deformation diagram [16].

Microstructural analysis of the region of concentrated deformation shows that plastic flow localization occurs successively at different levels [17]. At the microlevel it occurs as intra- and intergranular inhomogeneities [18], at the intermediate level it occurs as a consequence of a change-over from slip lines to slip bands or microbands to shear bands [19-21], and at the macrolevel it occurs as a Portwen-Le Chatelier effect [22] and Chernov-Luders lines [23], necking and failure. Consequently, it is possible to give an adequate description of plastic flow localization by means of such variables as stress, strain, and strain rate. Apart from these it is necessary to know the number, distribution, and rate of movement of existing inhomogeneities, i.e., defects.

Movement of defects occurs under the action of local stresses which may considerably exceed macroscopic stresses applied to a specimen. Inhomogeneity of the stressed state causes occurrence of different modes of deformation due to defects: translational, shear, and rotational [24]. Theoretical and experimental study of this effect has led to formation of the concept of structural levels of strain for solids [25]. According to this concept plastic deformation should occur at several structural levels each of which is characterized by a scale determined by the nature of structural defects responsible for shape change. Here the first is translational movement which always generates accommodational turning to another structural level. Difficulty in the relative movement of structural elements leads to interaction of them as a whole, occurrence of stress concentrators at the junction of several elements, and as a result of this to disruption of material continuity.

It is well known that the physical nature of damage may be entirely different. However, currently microcracks are acknowledged as a typical defect of the structure of solids. These defects are a typical type of inconsistency in solid material. Generation and development of inconsistencies is accompanied by processes of creep, plastic flow [26, 27], and conditions for forming macroscopic cracks are connected with microcracks [28, 29].

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Experiments show that microcracks in deformable metals and alloys are always observed in areas of strain localization: the boundaries of blocks, slip lines, and in areas with high dislocation density. The distribution of defect density over the length of fractured specimens is studied in [30, 31], and it is established that in the region of the neck the value of $\Delta\rho/\rho$ is higher than the average by 30%. Here sometimes not one but several zones of increased thinning are encountered [32].

Data for the kinetics of microdamage accumulation in the neck of an extended specimen provided in [33-38] show that after the start of plastic strain localization in the central part of the axial section microcracks and pores develop. The density of them increases with an increase in the level of strain. With further deformation there is merging of microcracks and pores into a central ductile separation macrocrack whose propagation leads to separation of a specimen into parts (macrofailure). Thus, the occurrence, growth, and coalescence of pores and microcracks are the basic effects with necking and it is necessary to include them in describing processes of localization.

In [39, 40] with the aim of considering changes in material structure a symmetrical tensor of the second rank $p_{ik} = n\langle s_{ik} \rangle$ is introduced determining the volume concentration and preferred orientation of macrocracks. Macroscopic parameter p_{ik} is obtained as a result of averaging for a statistical assembly of microcracks each of which is characterized by a microscopic tensor $s_{ik} = sv_i v_k$ (s is microcrack volume, v is vector of its orientation). In [39, 40] by means of nonequilibrium thermodynamic methods equations are constructed for the condition of solids with defects including a ratio of the relaxation type for stress tensor σ_{ik} and kinetic equations for the microcrack density tensor p_{ik} :

$$\sigma_{ii} = \xi e_{ii}^p - \alpha_2 \dot{p}_{ii}; \quad (1)$$

$$\Pi_{ii} = \alpha_2 e_{ii}^p - \alpha_1 \dot{p}_{ii}; \quad (2)$$

$$\sigma'_{ik} = L_{iklm}^{(1)}(p_{\alpha\beta}) e_{lm}^{p'} - L_{iklm}^{(2)}(p_{\alpha\beta}) \frac{\Delta p'_{lm}}{\Delta t}; \quad (3)$$

$$\Pi'_{ik} = L_{iklm}^{(2)}(p_{\alpha\beta}) e_{lm}^{p'} - L_{iklm}^{(3)}(p_{\alpha\beta}) \frac{\Delta p'_{lm}}{\Delta t}. \quad (4)$$

Here σ'_{ik} , $e_{lm}^{p'}$, p'_{ik} and $\sigma_{\ell\ell}$, $e_{\ell\ell}^p$, $p_{\ell\ell}$ are, respectively, traceless and isotropic stress tensor components, plastic strain rates, and microcrack density tensor; $\Delta(\dots)/\Delta t$ is tensor derivative with respect to time (the Yaumann derivative); $\Pi_{\ell\ell} = \partial F/\partial p_{ii}$ and $\Pi'_{ik} = \partial F/\partial p_{ik}$ are isotropic and traceless components of thermodynamic force Π'_{ik} operating on the system when the value of p_{ik} differs from equilibrium; F is free energy of a material with microcracks. Equations (1)-(4) are quasilinear, i.e., in the general case coefficients ξ , α_1 , α_2 , and $L_{iklm}^{(\alpha)}$ depend on invariants p_{ik} . Taking account of the symmetry of the microcrack density tensor the form of the dependence of kinetic coefficients $L_{iklm}^{(\alpha)}$ on p_{ik} is as follows [40]:

$$L_{iklm}^{(\alpha)} = l^{(\alpha)} \delta_{il} \delta_{km} + l_1^{(\alpha)} (p_{il} \delta_{km} + p_{kl} \delta_{im}) + l_2^{(\alpha)} p_{ik} p_{lm} \quad (5)$$

[$l^{(\alpha)}$, $l_1^{(\alpha)}$, and $l_2^{(\alpha)}$ are phenomenological coefficients].

It is noted that an attempt to define all damage and material by parameter p_{ik} is a significant assumption. Generally speaking an assembly of defects should be described by numerous statistical characteristics and above all by spatial spectra [41]. In this case a study of the evolution of defects leads to a set of kinetic equations for these statistical characteristics. A change-over from one form of defect to another should occur with disruption of stability for defect development and be accompanied by a sharp increase in their size. Description of deformation processes using the microcrack density tensor suggests that characteristic times for a change in it exceed the time for defect development for a finer structure.

On the basis of Eqs. (1)-(4) we consider effects of plastic flow localization with extension of a cylindrical specimen under conditions of uniaxial creep ($\sigma = \sigma_{xx} = \text{const}$). We shall proceed from a unidimensional scheme for the process; kinetic coefficients are assumed to be constant; irreversible strains obey the condition $\text{Spe}_{ik}^p = 0$, and the average stress is only determined by the elastic component of strain ($\sigma_{\ell\ell} = k U_{\ell\ell}$). Then the set of equations of state together with the mass conservation rule [42, 43] have the form

$$e_{xx} = \frac{2}{3l_1} \sigma_{xx} + \frac{2l_2}{3l_1} p_{xx}; \quad (6)$$

$$\dot{p}_{xx} = \frac{2l_2}{l_1(2l_3 + \alpha_1) - 2l_2^2} \sigma_{xx} - \frac{3l_1}{l_1(2l_3 + \alpha_1) - 2l_2^2} \Pi_{xx}; \quad (7)$$

$$\frac{\partial(S\rho)}{\partial t} + \frac{\partial(Sv_x\rho)}{\partial x} = 0, \quad (8)$$

where $e_{xx} = e_{xx}^e + e_{xx}^p$ is total strain rate (e_{xx}^e is elastic strain rate); $v_x = v_x(x, t)$ is the rate in a section with coordinate x at instant of time t ; $S = S(x, t)$ is the corresponding specimen cross-sectional area; $\rho = \rho(x, t)$ is material density. Since with uniaxial tension loosening of $\Delta\rho/\rho$ is in a physical sense the volumetric density of microcracks, then the change in ρ is clearly connected with parameter $p_{ik}(x, t)$ by the relationship

$$\rho(x, t) = \rho_0(1 - Sp_{ik}) \quad (9)$$

(ρ_0 is density at the initial instant of time).

As shown by the results of experiments for the change in defect density for deformed specimens obtained by the differential method of hydrostatic weighing [44], the maximum value of $\Delta\rho/\rho$ does not exceed 10^{-3} and the continuity equation may be written in the form

$$\frac{\partial S}{\partial t} + \frac{\partial(Sv_x)}{\partial x} = 0. \quad (10)$$

A consequence of the statistical description given for the behavior of an assembly of interacting microcracks [39] was the conclusion that the spectrum of possible reactions of solids on applying a load (absolutely unstable, metastable, equilibrium, i.e., curves 1-3 in Fig. 1) may be represented by a Landau expansion [45] for free energy [40, 46]

$$F = \frac{1}{2} Ap_{xx}^2 - \frac{1}{3} Bp_{xx}^3 + \frac{1}{4} Cp_{xx}^4 - D\sigma_{xx}p_{xx} \quad (11)$$

(A, B, C, D are phenomenological parameters for the material).

It is established in [27, 30] that the microcracks generated are extremely unevenly distributed through a deformed specimen; in surface layers their concentration is higher by one to three orders of magnitude than within the volume. With the aim of considering the spatially nonuniform situation with respect to p_{ik} we include in the expression for free energy a term which describes possible inhomogeneous distribution of the microcrack density tensor p_{ik} :

$$F = \frac{1}{2} Ap_{xx}^2 - \frac{1}{3} Bp_{xx}^3 + \frac{1}{4} Cp_{xx}^4 - D\sigma_{xx}p_{xx} - \frac{1}{2} D_1 \left(\frac{\partial p_{xx}}{\partial x} \right)^2 \quad (12)$$

(D_1 is a parameter depending on the state of the material structure). The equation for movement p_{xx} is obtained by substituting in (7) a partial derivative $\partial F/\partial p_{xx}$ by a variational derivative $\delta/\delta p_{xx} = \partial/\partial p_{xx} - \partial/\partial x \partial/\partial(\partial p_{xx}/\partial x)$:

$$\dot{p}_{xx} = \frac{2l_2}{l_1(2l_3 + \alpha_1) - 2l_2^2} \sigma_{xx} - \frac{3l_1}{l_1(2l_3 + \alpha_1) - 2l_2^2} \left(\Pi_{xx} - D_1 \frac{\partial^2 p_{xx}}{\partial x^2} \right). \quad (13)$$

After changing over to dimensionless variables $\Sigma = \sigma_{xx}/\sigma_s$, $\Gamma = e_{xx}t_m$, $\tau = t/t_m$, $\Pi = \Pi_{xx}/\sigma_s$, $a = d/d_0$, $\xi = \ell/d_0$, $p = p_{xx}$, $t_m = \ell_1/G$ we have

$$\Pi = A_1 p - B_1 p^2 + C_1 p^3 - D\Sigma; \quad (14)$$

$$\Gamma = A_2 \Sigma + A_3 \frac{dp}{d\tau}; \quad (15)$$

$$\frac{dp}{d\tau} = A_4 \Sigma - A_5 \Pi + A_6 \frac{d^2 p}{d\tau^2}; \quad (16)$$

$$\frac{da}{d\tau} = -\frac{1}{2} a\Gamma, \quad (17)$$

where $A_1 = A/\sigma_s$; $B_1 = B/\sigma_s$; $C_1 = C/\sigma_s$; $A_2 = 2\sigma_s/3G$; $A_3 = 2l_2/3l_1$; $A_4 = \frac{\sigma_s}{G} \frac{2l_2}{2l_3 + \alpha_1 - 2l_2^2/l_1}$; $A_5 = \frac{\sigma_s}{G} \frac{3l_1}{2l_3 + \alpha_1 - 2l_2^2/l_1}$; $A_6 = \frac{D_1}{d_0^2} \frac{3l_1 t_m}{l_1(2l_3 + \alpha_1) - 2l_2^2}$; d_0 and ℓ_0 are specimen gage length and diameter at the initial instant

of time; $d = d(x, t)$ is diameter in a section with coordinate x at instant of time t ; and G and σ_s are Young's modulus and specimen material yield strength.

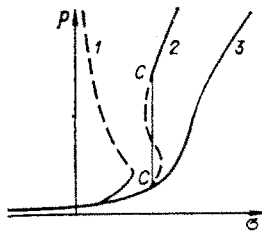


Fig. 1

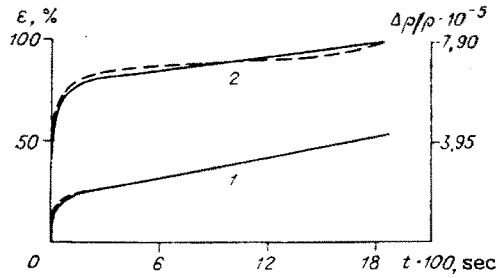


Fig. 2

A numerical study of set (14)-(17) was carried out using the finite difference method. Aluminum was selected as a specimen material which forms a well-developed neck and deforms under conditions of a strong increase (by two to three orders of magnitude) in true flow rates [47]. An estimate of constants in Eqs. (14)-(16) was made by the least squares method [48]. This was the basis for an approximation of creep curves (solid line 1 in Fig. 2) and defect density $\Delta\rho/\rho$ (solid line 2) obtained in the case of testing an aluminum specimen with $T = 18^\circ\text{C}$, $\sigma = 70\text{ MPa}$ [31]. Given in Fig. 2 are the results of numerical solution of set (6) and (7) (broken lines) for the following values of the parameters sought: $\ell_1 = 3.092 \cdot 10^{11}\text{ N}\cdot\text{sec}/\text{m}^2$, $\ell_2 = 1.519 \cdot 10^{15}\text{ N}\cdot\text{sec}/\text{m}^2$, $2\ell_3 + \alpha_1 = 8.027 \cdot 10^{10}\text{ N}\cdot\text{sec}/\text{m}^2$, $A = 7.23 \cdot 10^{18}\text{ Pa}$, $B = 9.438 \cdot 10^{22}\text{ Pa}$, $C = 4.069 \cdot 10^{26}\text{ Pa}$, $D = 2.616 \cdot 10^6$. It is noted that calculation coefficient ℓ_1 , which is an analog of material shear viscosity, almost coincides with experimental data presented in [49]. Values of the rest of the constants for an aluminum specimen are $\sigma_s = 35\text{ MPa}$, $G = 70\text{ GPa}$ [50].

Parameters A, B, C, and D, determined with treatment of experimental results, describe the stable reaction of the material to microcrack generation and growth which corresponds to material with a relatively small grain size. With the aim of considering property inhomogeneity, which is typical for actual metals and alloys, the statistical spectrum of solid reaction to crack formation was prescribed. Sets of constants for different dependences $p(\Sigma)$ were obtained by varying B with invariant A, C, and D.

The initial and boundary conditions for Eqs. (14)-(17) have the form

$$\begin{aligned} \Sigma = 1,44, p(\xi, 0) = p_0, d(\xi, 0) = 1, \xi_0 = 5, \\ \Gamma(\xi, 0) = 0, \nabla p(0, \tau) = 0, \nabla p(l, \tau) = 0. \end{aligned} \quad (18)$$

The results of numerical solution of set (14)-(18) for specimens with different original structure are presented in Figs. 3 and 4. In the region of material metastable reactions to microcrack generation and growth a plastic strain wave forms (Fig. 3). At instant of time $\tau = 7.6$ strain homogeneity is suddenly disrupted and local thinning appears in the specimen, i.e., a neck. The latter is reduced to a certain value after which it starts to propagate along the specimen until it embraces the whole of it. The plastic wave profile determining the form of the neck boundary propagates at a constant rate. Depending on the original state of the material with the presence of a spectrum of metastable reactions along the specimen it is possible for several sites of plastic strain localization to occur. Numerous necks were observed many times by experiment with tension under creep conditions [51] and with active deformation [16, 22, 23].

The reason for this behavior of specimens at a macroscopic level is existence of non-linear kinetics for development of a set of microcracks. The original statistical scatter of material properties leads to numerous kinetic changes with respect to variable p_{ik} . A consequence of a kinetic change is formation of a more ordered system of discontinuities providing the minimum plastic resistance over some slip direction. Here there is a sharp reduction in the time of stress relaxation, an increase in strain rate (Fig. 3a), and as a consequence plastic flow localization. This local increase in plasticity does not lead to specimen failure due to the finite jump with respect to parameter p_{ik} . From instant of time $\tau = 23.2$ in the region of the neck a deformation regime is established with a constant level of volumetric microcrack concentration and the value of Γ falls markedly.

The reason for neck propagation is inhomogeneous defect distribution forming a spatial front of a nonuniform structural (orientational) change. In sections embraced by a structural change intense plastic flow is realized accompanying localization. Existence of a stable reaction (so-called thermodynamic branches [40]) introduces a certain specific nature to the evolution of an assembly of defects and the features of strain localization. This is connected

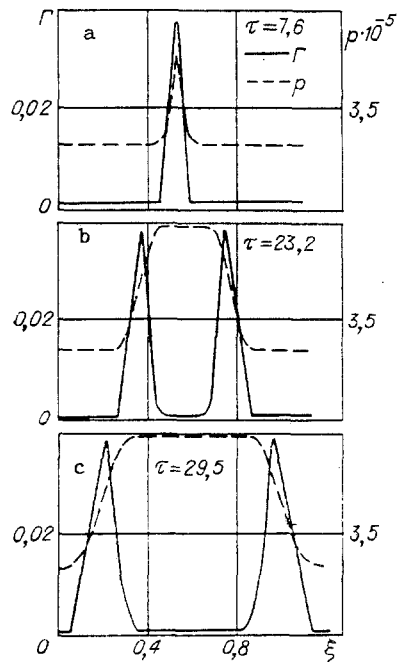


Fig. 3

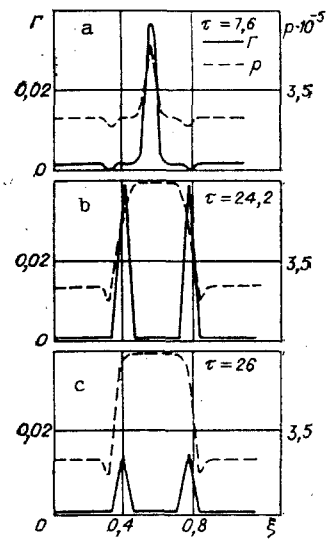


Fig. 4

with the fact that for fine-grained materials the process of microcrack growth is characterized by their relatively weak volumetric and orientation interaction, and the level of the applied stress is insufficient for a change into a metastable branch. Under these conditions plastic instability appears as occurrence of stabilization of local thinning in different areas of a specimen with a jumpwise change in the value of Γ and rebuilding of the material structure (see Fig. 4). It is natural to interpret this flow regime as unstable strain localization.

The case of an absolutely unstable reaction was studied in detail in [52]. Kinetics for the accumulation of discontinuities in this situation are characterized by explosive instability (aggravated regime [39]) and formation of failure sites. This occurs with coalescence of microcracks under conditions of a sharp increase in their concentration. At the macroscopic level this leads to an unlimited increase in strain rate and separation of a specimen into parts.

Thus, the approach developed makes it possible to model instability of the deformation process and plastic flow localization. The mechanism of plastic instability is an orientation kinetic change in parameter p_{ik} accompanied by space-time self-limitation of defects [53]. In this case a deformable material should be considered as a system in which during deformation dissipative structures arise with a capacity to accomplish more effectively macroplastic flow compared with movement of individual defects.

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CALCULATION OF THE EVOLUTION OF PLASTIC YIELDING AT THE TIP OF A CRACK
AND RELATED PHENOMENA

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The results of a computer calculation of the evolution of plastic yielding at the tip of a crack in a crystal are presented. The plastic yielding at the tip of the crack is due to thermally activated motion of dislocations in active slip planes of the crystal with the simultaneous action of an external tensile stress and thermal fluctuations. The plastic yielding and stress distributions at the tip of the crack at different instants of time are obtained. The effect of the plastic zone on the stress intensity factor (SIF) of the crack is calculated.

1. Introduction. In recent years microscopic models of the processes occurring in the neighborhood of a crack have become more and more widely used when investigating the mechanics of fracture. New ideas (the J-integral, the fine structure of the plastic zone, including the dislocation-free zone, etc.) have been proposed to describe them, while direct electron-microscope observations of the defect structure of the material in the region of the tip of the crack have, in turn, enabled the representation of the mechanics of fracture and the physical nature of the constants by which they operate to be refined.

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